

FREE SMOOTH \mathbb{Z}^p -ACTIONS ON THE THREE TORUS DIMENSIONAL

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ABSTRACT. We show that for a spectrally unitary \mathbb{Z}^p -action \mathbf{A} on the first homology group $H_1(T^3, \mathbb{Z})$ of the torus T^3 (i.e., 1 is an eigenvalue of $\mathbf{A}(\ell)$ for each $\ell \in \mathbb{Z}^p$), there exists a free \mathbb{Z}^p -action by real analytic diffeomorphisms on T^3 whose induced \mathbb{Z}^p -action on $H_1(T^3, \mathbb{Z})$ is \mathbf{A} .

1. Introduction.

Let p, q be natural numbers. We denote by a \mathbb{Z}^p -action of class C^r on the torus T^q a homomorphism φ of \mathbb{Z}^p into $\text{Diff}^r(T^q)$, where $\text{Diff}^r(T^q)$ is the group of C^r diffeomorphisms of the torus T^q , $0 \leq r \leq \infty$ or analytic. The \mathbb{Z}^p -actions on $H_1(T^q, \mathbb{Z}) \simeq \mathbb{Z}^q$ are closely related with the \mathbb{Z}^p -actions of class C^r on the torus T^q . In fact, any \mathbb{Z}^p -action φ of class C^r on the torus T^q induces \mathbb{Z}^p -action \mathbf{A} on $H_1(T^q, \mathbb{Z})$ called the *induced action* by φ .

Let \mathbf{A} be a \mathbb{Z}^p -action on $H_1(T^q, \mathbb{Z})$ such that \mathbf{A} acts by automorphisms of \mathbb{Z}^q . We call \mathbf{A} *spectrally unitary* if 1 is an eigenvalue of $\mathbf{A}(\ell)$ for each $\ell \in \mathbb{Z}^p$.

We recall that a \mathbb{Z}^p -actions φ of class C^r on the torus T^q is free if for each $\ell \in \mathbb{Z}^p - \{0\}$, $\varphi(\ell)$ has no fixed points. In particular, a free \mathbb{Z}^p -action does not have finite orbits. It follows from the Lefschetz fixed point theorem that a free \mathbb{Z}^p -action φ on the torus T^q induces a spectrally unitary \mathbb{Z}^p -action on $H_1(T^q, \mathbb{Z})$.

Theorem 1. *Let \mathbf{A} be a spectrally unitary \mathbb{Z}^p -action on $H_1(T^3, \mathbb{Z})$. Then there is a free real analytic \mathbb{Z}^p -action on the torus T^3 whose induced action on the first homology group of T^3 is precisely \mathbf{A} .*

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